Outliers in Time and Frequency Measurements

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ABSTRACT

This paper describes methods for dealing with outliers in time and frequency measurements. It shows how bad measurements can occur, what characteristics they can have, and how outliers can be detected, analyzed and removed. The paper includes examples of outlier analyses using the Stable32 program for frequency stability analysis.

INTRODUCTION

Any measurement is subject to anomalies due to either the subject of the test or the measurement system. In the case of precision time and frequency measurements, the results can have extreme values due to defects in the device under test or be contaminated by problems with the reference source, the measuring system or its environment. When such anomalies occur, it is important to detect and quantify the outlier data, and identify the source and reason for the problem.

This discussion of outliers in time and frequency measurements applies to phase data in the form of x(t) with units of seconds or dimensionless fractional frequency data y(t) for frequency sources (clocks and oscillators) having moderate to high stability. The terms clock, oscillator, frequency source and device under test are used interchangeably. Clock measurements are preferably made as phase (time error) data because that can be more generally used to obtain frequency data than vice versa. These data are assumed to be equally spaced at an appropriate measurement interval and with sufficient resolution to support the precision needed for the sources being measured.

OUTLIERS

Outliers, in this context, are results of a clock measurement that depart from the normal range of scatter due to the expected noise sources of the associated devices [1]. In other contexts, outliers may represent exceptionally good cases (e.g., high intelligence or skills [2]), but herein they are bad. Nevertheless, like death and taxes, outliers cannot entirely be avoided, and means must be devised to detect, quantify, identify, explain and hopefully reduce them. Often, outliers must be removed from phase and frequency data before it can be analyzed. Then, the outliers should be revisited, at a minimum to document and justify their removal.

In some cases, such as anomaly debugging and transient radiation testing, the outlier represents the desired data. But normally an outlier is undesirable and must be detected, removed, explained and its cause hopefully eliminated.

Outliers are also considered to mean transient disturbances (e.g., spikes) not permanent changes (e.g., steps) or periodic variations (e.g., interference) in frequency. These may, however, comprise multiple disturbances (e.g., doublets or bursts, see Outlier Pattern below). The analysis of frequency steps, jumps and cyclic variations are other topics not considered in this paper [3-5].

• Outlier Types

Outliers can be associated with either phase or frequency data but are most readily observed as the latter because they will show sharp excursions from the normal scatter. The frequency excursion can be in either direction, and its size can range from slightly above or below normal to a very large departure from normal. The corresponding change in phase slope can be harder to discern and quantify, so outlier detection is usually applied only to frequency data.

• Outlier Detection

The most obvious way to find outliers in a data set is to visually examine the data. While somewhat subjective, this is very effective and should be the first step in any outlier detection process. Not only does visual inspection show any outliers but it also provides additional insight regarding their relative size, polarity and spacing.

Automatic outlier detection requires a robust means to compare a data set against some measure of its normal scatter. That gauge has to itself be immune to any outliers, so, for example, one would use the median value rather than the average which would be affected by an extreme value.

The median absolute deviation (MAD) is a robust means of outlier recognition [6-9] based on the median of the data. It is the median of the scaled absolute deviations of the data points from their median value and is defined as:

$$MAD = Median\left[\ \left|y_i - m\right| / 0.6745\right]$$

where m=*Median* [y_i], and the factor 0.6745 makes the MAD equal to the standard deviation for normally distributed data.

As an example, a 10,000 point set of simulated white FM noise with a nominal ADEV of 1.0 and no outliers generated by the Stable32 program [10] had a normally-distributed histogram, a standard deviation of 0.997, an Allan deviation of 1.002 and a mediation absolute deviation of 0.979.

To find outliers, each frequency data point, y_i, is compared with the median value of the data set, m, plus or minus the desired multiple of the MAD. A sigma factor of 5 is satisfactory for identifying outliers in most cases, and that is the Stable32 default.

• Outlier Size

Outliers can be roughly categorized into two sizes, moderate and gross. Moderate outliers will show up on a data plot as larger than normal excursions where the “good” data are still visible, while gross outliers are so large that the normal data are seen only as a line. Moderate outliers are of particular concern because they may indeed represent valid data and require judgment to reject them as bad measurements. Their designation may depend critically on the outlier detection method and criterion. Gross outliers are obviously bad and must be removed from the data set before further analysis can be accomplished.

• Outlier Patterns

Frequency outliers are associated with certain phase and frequency patterns as shown in Figure 1. These patterns have characteristics such as size and polarity, single, double or burst occurrences. They may be common or rare and occur randomly or periodically, and may or may not be correlated with some other event. Examination of these outlier patterns can provide insight into their cause.

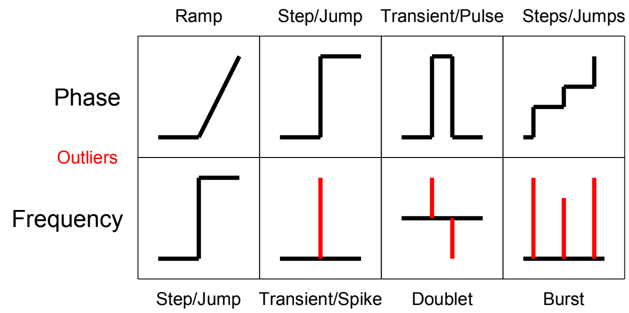

Figure 1. Outlier Patterns

Consider the fundamental relationship that frequency is the rate of change of phase. Then, for phase data, an abrupt change in phase slope corresponds to a step in source frequency, a step in phase corresponds to a spike in measured frequency, and a transient phase excursion is associated with a doublet frequency response. Thus a step change in source frequency does not result in an abrupt phase change or a frequency outlier, but a step phase change does produce a frequency outlier and a phase pulse produces a pair of equal and opposite frequency outliers. Bursts of phase steps/frequency spikes can also occur. The cause of these outlier patterns is not necessarily within the various devices but may be due to their interconnections. Fast phase steps or spikes could be caused by a mechanical disturbance to the clock connection. Note that "fast" is relative to the measurement interval. Even a slow phase change will make a frequency spike at a slow measurement rate. Phase ramps can be caused by gradual temperature change affecting the clock connecting cable and appear as a pseudo frequency change. A permanent phase step is arguably worse than a transient phase pulse, especially for a clock.

The idealized patterns of Figure 1 can be moderated to some extent in the real world. For example, the edges of a phase step or pulse may not be extremely abrupt and, because the associated frequency excursion depends on the rate of change of phase, it will not be as extreme. At some point, one considers the phase change as a ramp which then corresponds to a frequency offset rather than an outlier, and a smoothed series of phase steps becomes frequency pulse. It's really a matter of degree, possibly determined by the system bandwidth. A slow transient frequency excursion can cause a permanent clock error without being a significant outlier.

See Table I for a list of phase and frequency outlier characteristics for XO, passive AFS and PLL devices.

• Outlier Removal

Outliers may need to be removed from the raw data in order to proceed with their analysis. Because the clock data are a time series, a gap must be inserted in place of the outlier to maintain the correct time relationship. A zero as often used as this place filler, and it is especially convenient for fractional frequency data because that does not affect most stability calculations. If necessary, a valid fractional frequency value of 0 can be replaced by a very small value (e.g., 1e-99) to distinguish it from a gap. Zero is also often a valid phase value at the start and end of a data set.

• Outlier Causes

Outliers can be caused by any part of the measurement system including (1) the source under test, (2) the reference, (3) the connections to the measuring instrument, (4) the measurement instrument, (5) the data storage means, (6) the power supplies, and (7) the operating environment.

• Outlier Cases

Consider, for example, the case of a frequency source (e.g., quartz crystal oscillator or rubidium frequency standard) being measured by a dual mixer time difference clock measuring system versus a "perfect" reference (e.g., hydrogen maser) as shown in Figure 2.

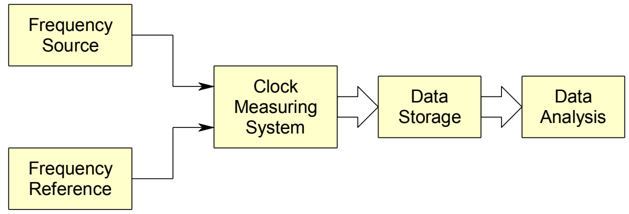

Figure 2. Basic Clock Measuring Setup

If the source should experience a sudden frequency change (a step in frequency) the phase slope would immediately change; this would not produce a frequency outlier but rather a permanent frequency jump to a new value. If, however, the phase of the signal should abruptly change, say from a mechanical disturbance that altered the length of its RF output connection by 10 mils or about 1 ps, then there would be a momentary change in the rate of change of phase (i.e., the apparent frequency), over 1 second of about $1x10^{-12}$. If the normal noise level was below that, there would be a small frequency outlier for that 1 second measurement interval. Suppose instead that for some reason the measuring system missed one of the 10 MHz zero crossings from the source. Then there would be a huge frequency outlier, 100 ns over 1 second or 10^{-7}. One should always be alert for phase changes that correspond to the RF carrier period or multiples thereof. Otherwise, a relatively large phase step is quite unusual for either a crystal oscillator or rubidium frequency standard, and one has to consider the possibility of a transient frequency over the measurement interval. For example, if the RFS should momentarily loose its internal frequency lock, there would be a resulting frequency outlier. An event of that type is best diagnosed by both clock and monitor data with the shortest possible measurement interval which might resolve the characteristics of the frequency transient.

Now consider a more complex example of a frequency source followed by a phase locked crystal oscillator as shown in Figure 3. Besides the possibility of disturbances to the overall output signal, the most likely causes for a frequency outlier are a frequency transient of the frequency source (tracked by the PLL) or a phase transient of the XO or PLL.

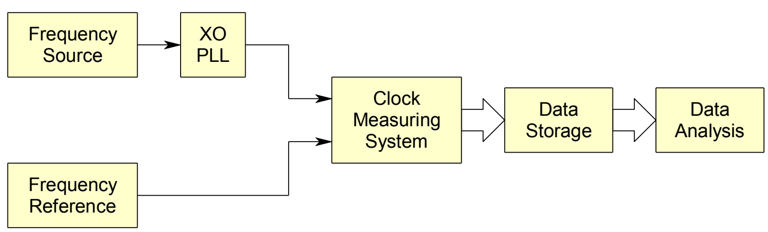

Figure 3. PLL Clock Measuring Setup

One has to consider the dynamics of the PLL when observing the response over short measurement intervals. The PLL tends to filter out fast disturbances of the frequency source and directly pass phase perturbations of the PLL locked oscillator or its loop components. Thus a phase step or transient is more likely caused by the PLL section, the former producing a single frequency outlier and the latter a doublet. Conversely, the PLL follows slower frequency changes of its reference frequency source and removes those of its locked oscillator. A phase step of a free-running or passive AFS XO will persist while that of a locked XO will be removed by the PLL response, generally a damped transient ringing that reduces the phase error to zero.

Table I summarizes the characteristics of some common outlier mechanisms for crystal oscillators (XO), passive atomic frequency standards (AFS) and a phase locked crystal oscillator (PLL).

Table I. Outlier Characteristics					
Device	Disturbance	Outlier(s)		Remarks	Example
		Phase	Frequency		
Frequency Source (XO or AFS)	Frequency Step	No (Slope Change)	No (Step)	Permanent frequency step changes phase slope but does not cause phase or frequency outlier.	XO crystal frequency jump.
	Frequency Transient	No (Step)	Yes (Spike)	Transient frequency change is a frequency outlier and causes a permanent phase step.	AFS servo loop disturbance.
	Phase Step			Permanent phase step is not a phase outlier but causes a frequency spike.	RF O/P cable length change.
	Phase Transient	Yes (Spike)	Yes (Doublet)	Transient phase change causes a pair of opposite polarity frequency outliers.	Loose RF output connection.
Phase Locked XO with AFS Reference	Reference Frequency Step	No (Smoothed Slope Change)	No (Smoothed Step)	Reference frequency jump is filtered by PLL to slower change w/o O/P phase or frequency transients.	RFS lamp jump.
	Reference Frequency Transient	No (Smoothed Step)	Maybe (Smoothed Transient)	Transient reference frequency change is smoothed by PLL response.	AFS XO frequency jump.
	Reference Phase Step	No (Step)	Yes (Smoothed Spike)	Reference phase change is tracked, smoothed by PLL response.	Reference phase change
	Reference Phase Transient	Yes (Spike)	Yes (Smoothed Doublet		Reference phase disturbance
	PLL Phase Step	No (Step)	Yes (Spike)	PLL can experience a permanent phase change.	PLL phase detector jump.
	PLL Phase Transient	No (Step)	Yes (Spike)	PLL will correct XO phase error as loop recovers.	PLL XO phase pop.

For the case of two frequency sources, the unit under test and the reference, outliers can be caused by either and are indistinguishable except for their relative sense. This includes their power supplies, cables, environments and any other components up to the measuring system. The best way to exclude a problem with the reference is to simultaneously measure several sources. For a single channel system, substituting different units under test can be a useful diagnostic. Slowly varying environmental factors like temperature usually do not cause outliers, but mechanical shock and vibration certainly can, along with such events as EMI and power line transients (e.g., thunderstorms, fluorescent light ballast turnoff). Acoustic noise can affect frequency sources that employ critical optics. Power line hum and ripple, ground loops, other frequency sources, etc. may cause erroneous measurements and periodic interference but are seldom the cause of discrete outliers. For multichannel measuring systems, it can sometimes help to use two channels for the measurement to rule out a problem with one input of the measurement hardware. Some clock measuring systems experience invalid results when the input signals are at or near phase coincidence. The data transfer, storage and analysis hardware/software is seldom a cause of outliers, but other analog and digital portions of the measurement hardware certainly can be, especially due to external power line transients and the like. Most of the time, for a well-established measurement setup, the problem will lie within a particular source under test, and that can often be confirmed by continuing or repeating the test along with substituting another similar device for comparison. Transient radiation testing can, of course, cause large phase and frequency excursions but in that case the "outliers" represent the desired data.

There are many possible causes for a phase step, and some of the most common are as follows:

1. An abrupt change in cable or interconnection length, perhaps due to a bad contact or connector, because of a mechanical shock or other physical disturbance.
2. Acceleration applied to a quartz crystal oscillator.
3. An abrupt detuning of an RF output amplifier tank circuit or filter, perhaps due to a bad connection or defective component.
4. A transient radiation pulse affecting a phase-sensitive circuit such an analog comparator, digital counter or phase detector.
5. A power supply glitch affecting a phase-sensitive circuit.
6. Disturbance of a PLL phase detector or associated circuits.
7. A crystal oscillator “phase pop”.
8. Glitches, missing counts, upsets, etc. in a digital counter or divider.
9. A large, fast, brief frequency change.

• Outlier Forensics

It is usually important to identify the cause of an outlier, especially if it could have been due to the clock under test. It is easier to diagnose the cause if there are multiple clocks under test because if the data for the other clocks are normal then one can presume that the reference and most of the measuring system was OK. Supplementary information such as monitor signals can be helpful in determining whether the clock under test experienced a problem. Timetags can be a valuable tool to correlate the outlier with other measurements and events such as power line glitches. Periodic outliers often point to external environmental effects such as temperature cycling and human activity; weekly disturbances are almost always manmade. Rare outliers are particularly hard to pin down. Nevertheless, one should always assume that there is a reason for every outlier; ignoring them can bring future grief. One generally sees more outlier problems with unpackaged experimental hardware than finished products. Effort put into making the measuring system robust (uninterruptable power, high quality cables and connectors, redundant references and power supplies, continuous data backup, etc.) pays off in the long run. Outlier investigation is aided by having a suite of analysis tools well-suited to that purpose. One needs to be able to easily plot clock and monitor data, overlay them in time, and compare them against other clock records, looking for patterns and correlations.

• Outlier Analysis

Outliers can be analyzed by many of the same tools as normal data. In fact, for certain debugging activities, the outlier may actually represent the data of interest. A plot of the raw data can provide an immediate visual indication of the outlier size, polarity and periodicity. A list of the outlier values and positions (data point numbers or timetags) can be used to analyze these properties in more detail.

Figures 4-8 show an example of frequency data having outliers, how they can be plotted and detected using the Stable32 program for frequency stability analysis [10], and how information regarding them can be exported to a spreadsheet program like Microsoft Excel® for further analysis.

An example of a 1000-point set of frequency data with three large outliers is shown in Figure 4. Visual inspection makes it clear that the outliers are much larger than the normal median (≈ 0.48) and scatter (white FM noise with ADEV ≈ 0.29 w/o outliers, largest outlier ≈ 665), that they have different sizes, the same polarity and no particular spacing. The data points have a certain sampling interval (1 second in this case) and, if there are timetags, they can be associated with calendar dates and times.

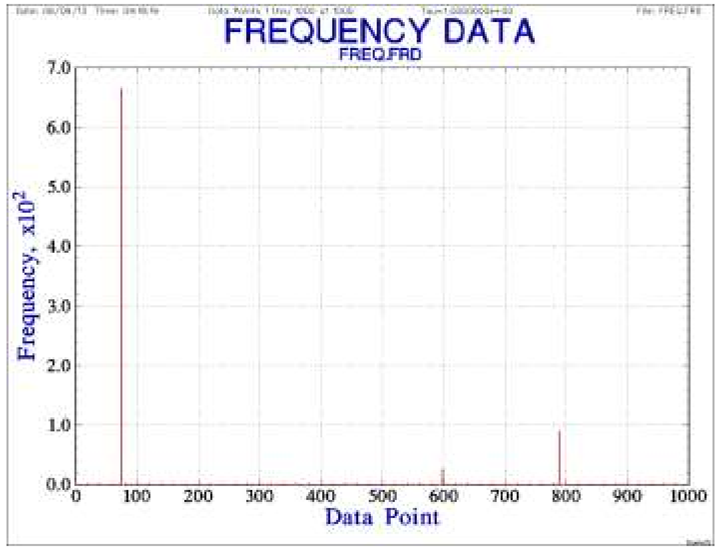

Figure 4. Example of Frequency Data with 3 Outliers

Stable32 includes a Check function that can check frequency data for outliers exceeding a certain MAD threshold as shown in the example of Figure 5. The median absolute deviation for the complete 1000 point data set is about 0.37 (a robust form of standard deviation) and the sigma factor is set to 5 times that as the outlier threshold. Three outliers are detected, and may optionally be removed to facilitate further analysis.

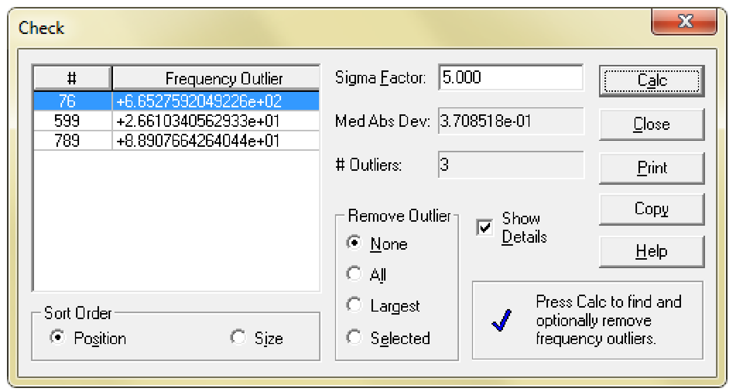

Figure 5. Example of Stable32 Check Function Screen

Stable32 Version 1.60 and higher includes provisions for writing an Outlier.dat file that contains the results of a Check function outlier analysis. It lists general information about the analysis followed by the data point number, MJD timetag (if available) and outlier fractional frequency value for each outlier detected as shown in Figure 6. The usual options for MAD threshold and sort order apply.

```
FREQUENCY OUTLIERS FOR FILE: FREQ.FRD
Points 1 thru 1000 of 1000
Median Absolute Deviation: 3.708518e-01
Sigma Factor: 5.00000e+00
# Outliers:   3
#        MJD              Frequency
000076   56446.00086806   +6.6527592049226e+02
000599   56446.00692130   +2.6610340562933e+01
000789   56446.00912037   +8.8907664264044e+01
```

Figure 6. Example of Stable32 Outliers.dat File

The Outlier file can be read into Excel for further analysis using that program's general space delimited format. The MJD timetag can be converted to an Excel date number by subtracting 15,018 and then displayed as a calendar date and/or time of day as shown in Figure 8. The outliers can then be plotted as a function of data point # (see Figure 7) or date/time and examined for such characteristics as size, periodicities and clusters or plotted against some other time series to look for correlations.

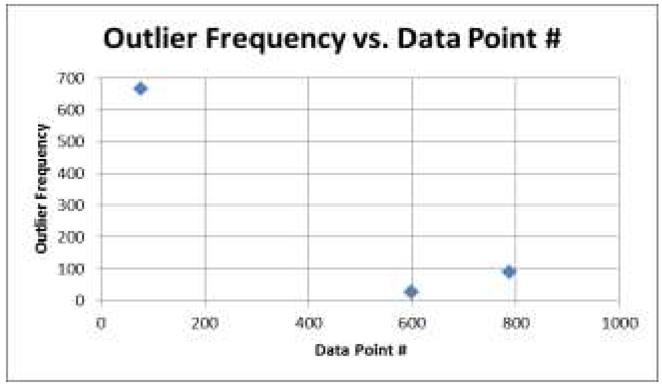

Figure 7. Excel Outlier Plot

FREQUENCY	OUTLIERS	FOR	FILE:	FREQ.FRD	
Points	1	thru	1000	of	1000
Median	Absolute	Deviation:	3.71E-01		
Sigma	Factor:		5.00E+00		
#	Outliers:		3		
#	MJD	Frequency	Excel Date #	Date	Time
76	56462.00087	6.652759E+02	41444.00087	06/19/13	0:01:15
599	56462.00692	2.661034E+01	41444.00692	06/19/13	0:09:58
789	56462.00912	8.890766E+01	41444.00912	06/19/13	0:13:08

Figure 8. Outlier Data Imported into Excel

• Outlier Data

Figure 9 shows an actual phase record of an ovenized crystal oscillator versus a small rubidium oscillator [11]. These data contain an abrupt phase jump of about +95 ps at the center of the record. Closer examination with the zoomed Statistics function plot shows that this phase step took place over a single 10 ms measurement interval (a similar observation at a 1 ms measurement interval showed a phase step that took about that long, a value consistent with the oscillator Q). The corresponding frequency record is shown in Figure 10. It shows the expected positive frequency spike of $95\text{x}10^{-12}/1\text{x}10^{-2} = 9.5\text{x}10^{-9}$. This is a fairly common type of outlier and can be caused by mechanically disturbing the cables connecting the sources to the measuring system or internally in the crystal oscillator. In this example, the crystal oscillator is prone to "phase pops", which are most likely caused by its crystal resonator. They tend to cluster together although it can go for long periods without such anomalies. The fast phase jumps would affect the output short term stability whether free-running, in a passive AFS frequency lock loop or as the locked oscillator in a phase lock loop. The Check function detects this frequency outlier at point # 2306 as having a value of about $+9.45\text{x}10^{-9}$, x63 larger than the MAD = $1.50\text{x}10^{-10}$. If the phase had taken several measurement periods to complete its jump, the frequency record would have shown several smaller contiguous outliers. If the phase had jumped back to near its original value, the frequency record would have shown a "doublet", a second negative spike of about the same size. If the phase had taken several positive jumps close together, the frequency record would have shown a burst of separate outliers. The crystal oscillator frequency was adjusted to be very close to that of the rubidium reference, and the phase changed only about -25 ps over the 40 second record, indicating a frequency difference of only about $-6.2\text{x}10^{-13}$. If the frequency outlier is removed, the combination of the two sources has a predominately white FM noise characteristic with a 1-second stability of about $6.9\text{x}10^{-12}$, determined mainly by the rubidium frequency standard.

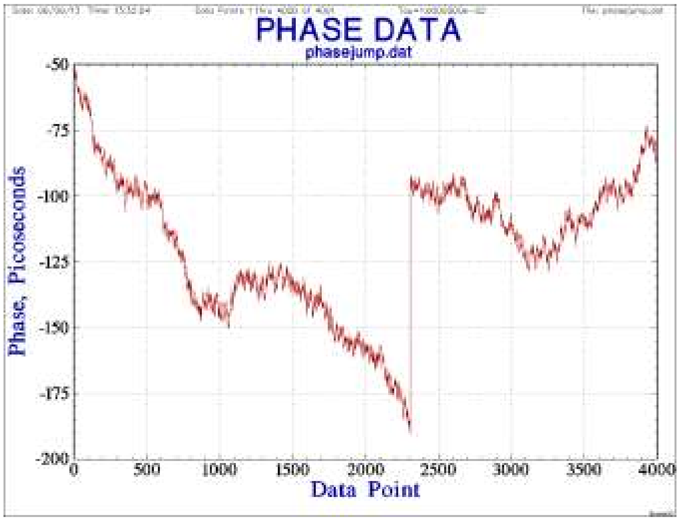

Figure 9. Phase Record with Jump

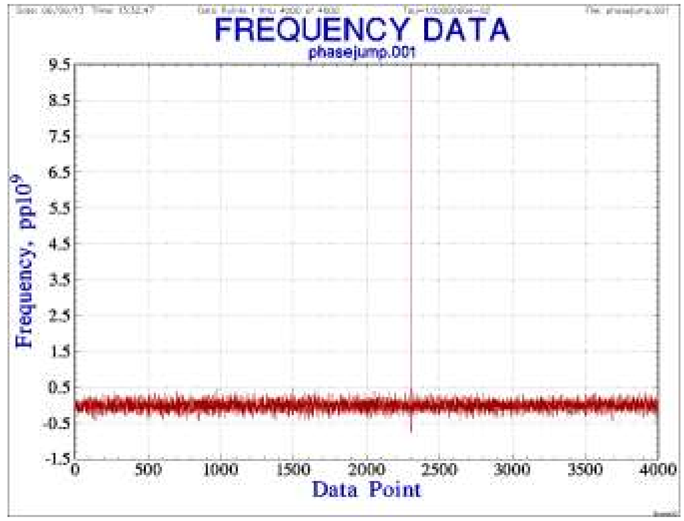

Figure 10. Frequency Record with Spike

• Outlier Simulations

The Stable32 Noise function can be used to generate simulated phase and frequency data with various types of outliers. The first simulation in Figures 11 and 12 starts with 500 points of simulated 1 second phase data having a $1\text{x}10^{-11}$ noise level to which is added a relatively large 1 ns positive phase step by selecting half of the phase data and using the Scale function to append $1\text{x}10^{-9}$ and then converting it to frequency data to see the resulting frequency outlier.

The second simulation in Figures 13 and 14 does the same thing except that two opposite phase steps are added to produce a phase pulse and a pair of "doublet" frequency outliers.

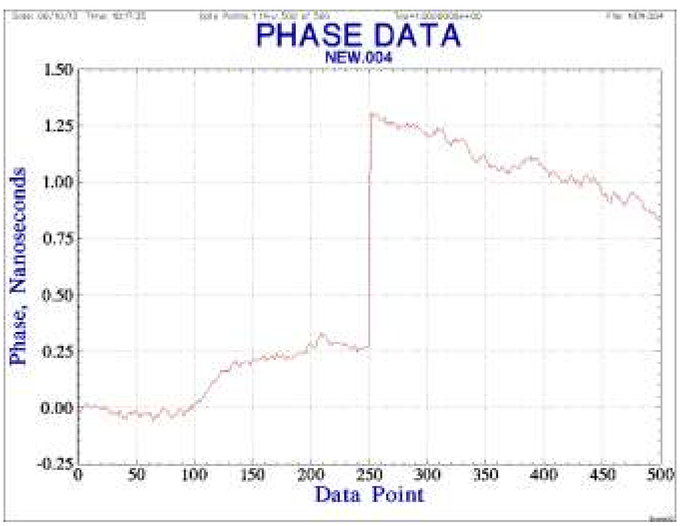

Figure 11. Simulated Phase Data with Step

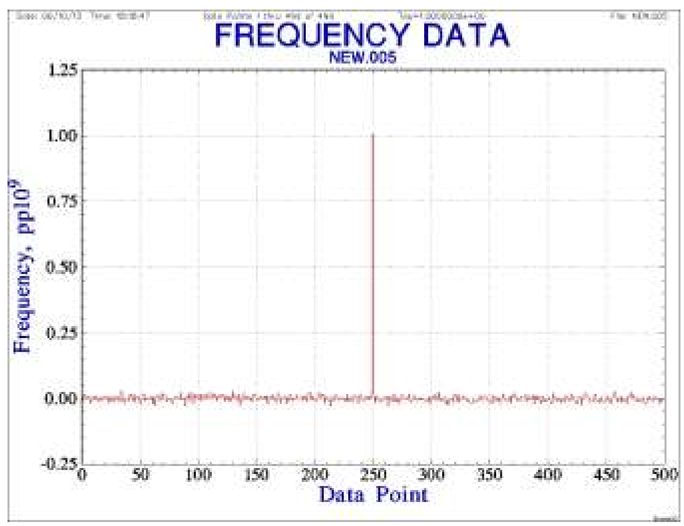

Figure 12. Simulated Frequency Data with Spike

Figures 11 and 12 show simulated phase data with a phase step and the corresponding frequency data with a frequency spike (outlier). The phase data is 500 points of 1-second white FM noise with a nominal 1-second ADEV of $1\text{x}10^{-11}$. It has a p-p phase variation of about 350 ps and an actual ADEV = $1.018\text{x}10^{-11}$ to which is added a +1 ns step at the center. The corresponding frequency data has a spike of $+1\text{x}10^{-9}$. This outlier is reported as $+1.0060\text{x}10^{-9}$ ns at point 250.

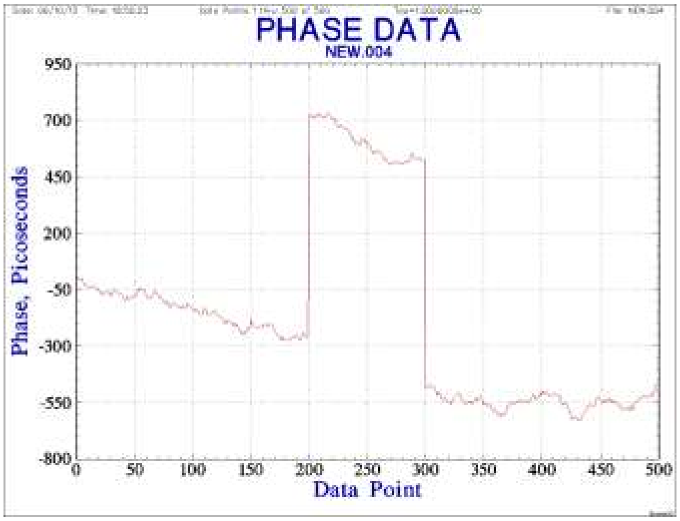

Figure 13. Simulated Phase Data with Pulse

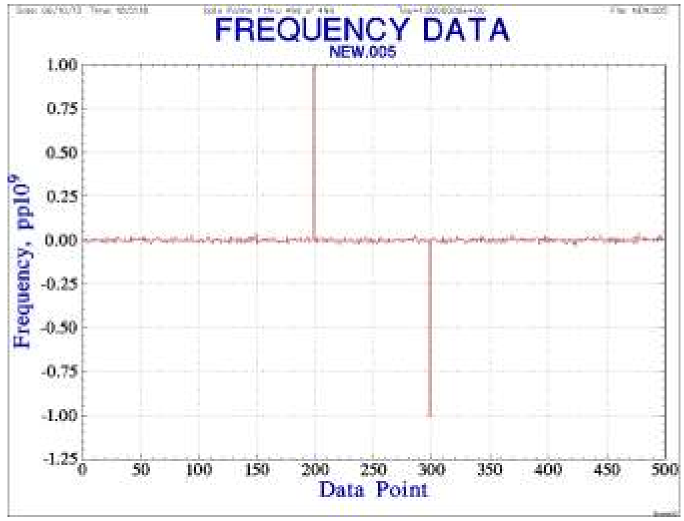

Figure 14. Simulated Frequency Data with Doublet

Figures 13 and 14 show simulated phase data with a phase pulse and the corresponding frequency data with a frequency doublet (2 opposite outliers). The phase data are 500 points of 1-second white FM noise with a nominal 1-second ADEV of 1×10^{-11}. They have a p-p phase variation of about 500 ps and an actual ADEV = 1.010×10^{-11} to which steps of +1 ns and -1 ns are added at points 40% and 60% of the data. The corresponding frequency data have spikes of $\pm 1 \times 10^{-9}$. These outliers are reported as $+0.984 \times 10^{-9}$ at point 199 and -1.008×10^{-9} at point 299.

• Outlier Modeling

It is seldom possible to predict how a device misbehaves to produce a specific outlier, but it is often feasible to model its response to a certain stimulus such as a power supply glitch, and to test the model by inducing the effect in actual hardware (e.g., switching a small capacitor across the output of a resistive source to produce a phase step, or using a DDS synthesizer to make precise instantaneous phase changes). Such modeling is very device-specific and cannot be generalized here, but examples can be found in the literature of clock analysis, i.e., analyses performed on the time keeping system of GPS navigation payloads in response to various disturbances. Such analyses can be usefully applied to clock systems employing phase locked oscillators and other elements to relate their response to internal disturbances to observed measurements. In this way, measurement outliers can be better explained in terms of physical phenomenon.

• Outlier Examples

As an example of actual clock measurement data having frequency doublet outliers, a 5 MHz DDS function generator [12] was set to produce 90° binary phase shift keying (BPSK) at 10 second intervals, and phase data were collected for 2 minutes at a 0.1 second measurement rate. The expected phase (time) deviation is ±50 ns (one-quarter of the 200 ns carrier period) and the expected fractional frequency deviation is 5×10^{-7} (50 ns divided by the 0.1 s measurement interval) as shown in Figures 15 and 16. The actual deviations are somewhat irregular because the modulation and measurements are asynchronous. The average slope of the phase data represents the nominal frequency offset ($\approx -1 \times 10^{-8}$).

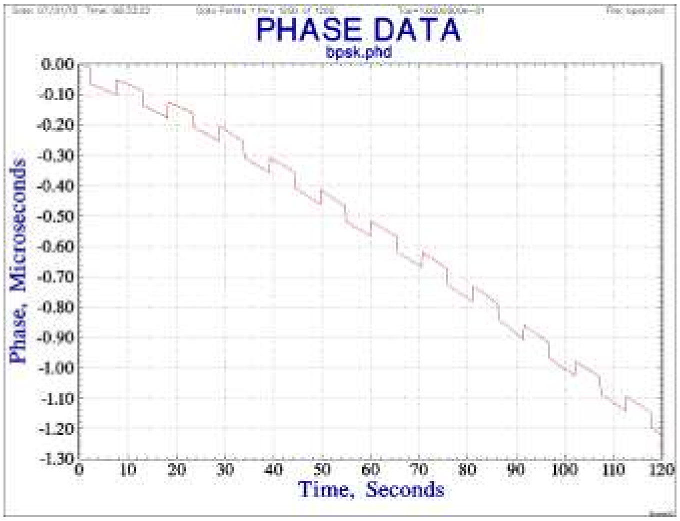

Figure 15. BPSK Phase Data

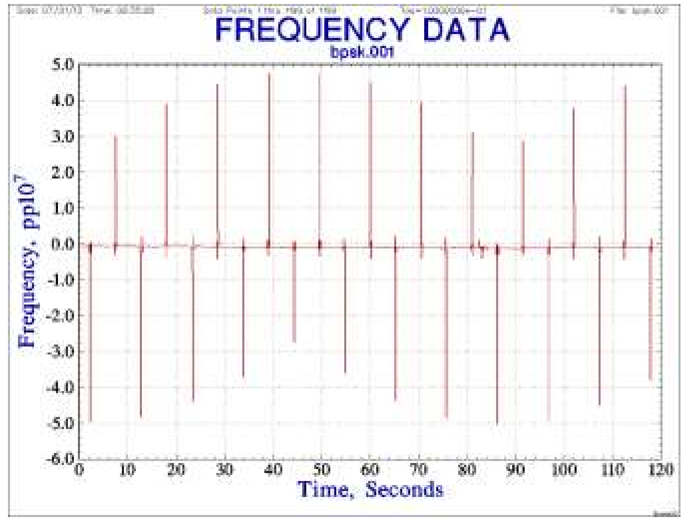

Figure 16. BPSK Frequency Data

Another example is produced by switching a 270 pF capacitor across the output of a matched 50 ohm 10 MHz source and load to produce a phase step of about 10 ns, either directly (Figures 17 and 18) or via the reference input to a phase-locked loop (Figures 19 and 20), while making phase measurements at a 1 ms sampling rate. The response time of the PLL smooths the phase transitions, thereby making smaller but longer frequency outliers. Note that the capacitor is switched manually for the two separate runs so their timing is not identical.

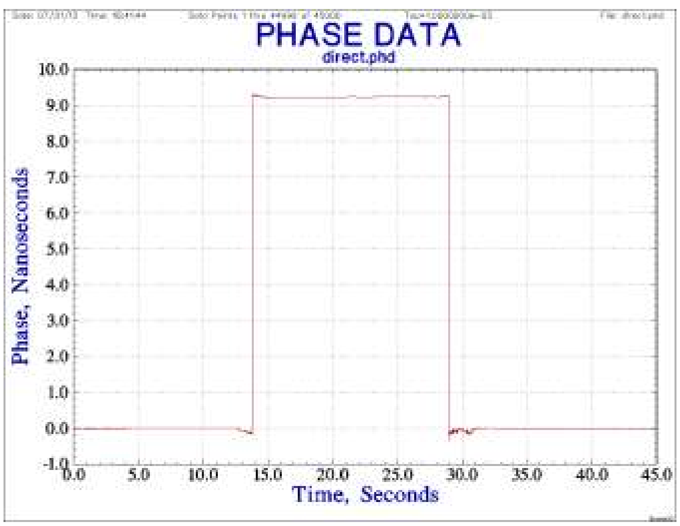

Figure 17 Direct Phase Data

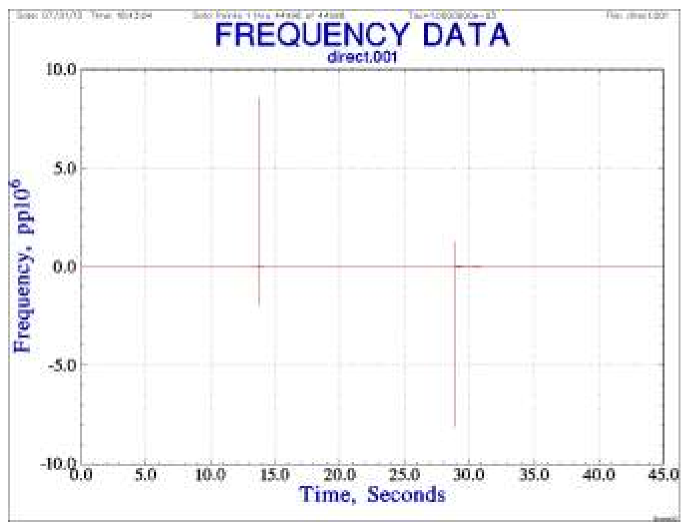

Figure 18 Direct Frequency Data

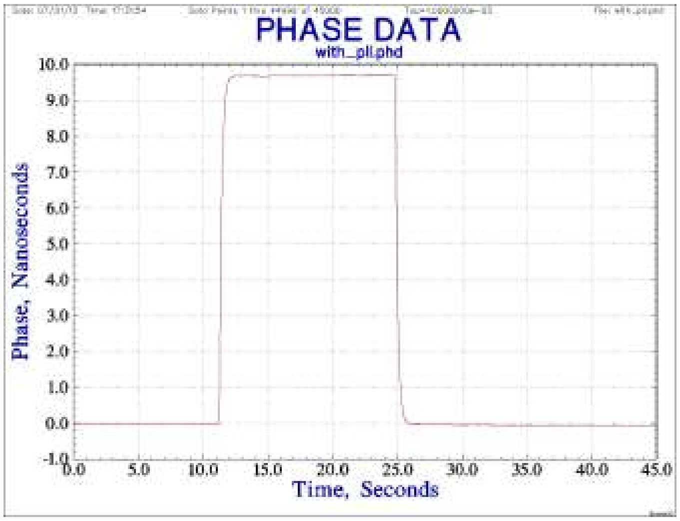

Figure 19 PLL Phase Data

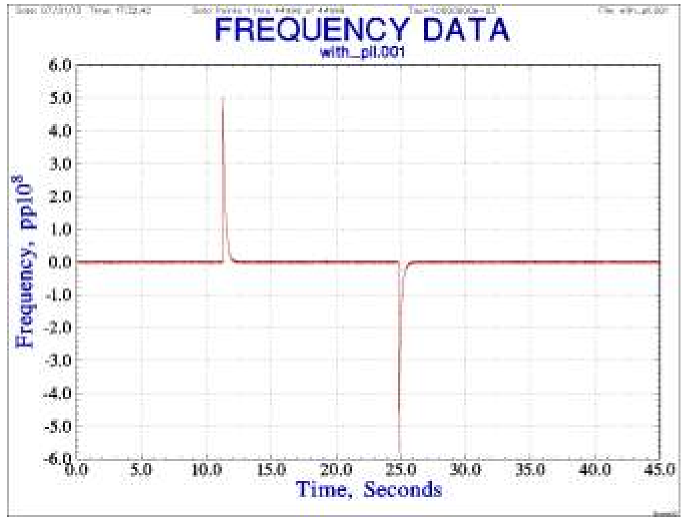

Figure 20 PLL Frequency Data

The phase transition at the output of the PLL requires about 0.45 second, as shown in Figure 21, in agreement with the PLL bandwidth of 0.5 Hz (f=0.5 Hz, ω=3.14 rad/s, τ=0.318 s, 68% of 0.45 s = 0.31 s). In the steady state, the 1st order PLL tracks the phase of its reference input with a small phase offset and no frequency error. The bipolar frequency spikes are reduced by two orders of magnitude from about $8x10^{-6}$ to $6x10^{-8}$ but still dominate the frequency record, and the size of the frequency outliers for an instantaneous phase step is determined by the PLL response rather than the sampling interval. Each PLL frequency outlier comprises about 775 1 ms samples.

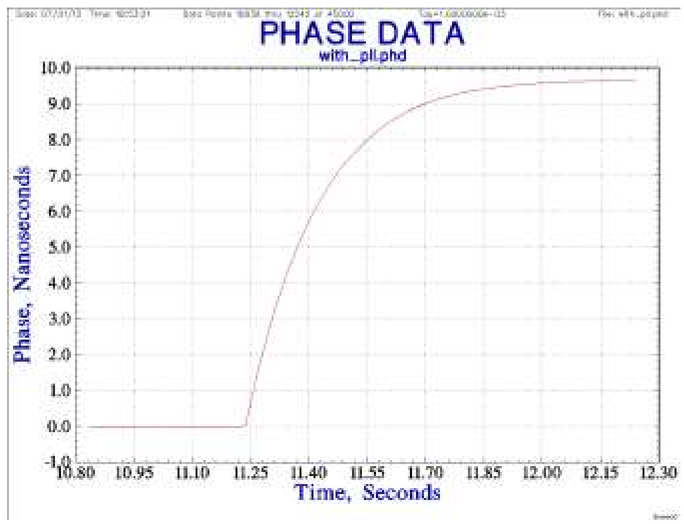

Figure 19 PLL Phase Transition

CONCLUSIONS

Outliers are unavoidable in clock measurements although every effort should be taken to minimize those due to the test setup, measurement system and frequency reference. Those that remain are then caused by the unit under test and therefore must be detected, analyzed and understood. While those outliers may have to be removed from the data set to allow further analysis, they still need to be investigated with the goal of determining their root cause and eliminating it. Outlier investigation requires careful detective work supported by patience and good analysis tools. Experience leads to the generation of a mental check list of possible causes for bad measurements based on the characteristics of the outliers and the device under test. Comparisons with similar devices can provide insight. The crucial question is often whether the anomaly is due to the device under test or the measurement system. Modeling and simulations may help to verify potential explanations, particularly if complex control loops are involved. But even with the best test methodology and anomaly investigation, questions may remain, especially when such occurrences are rare. Nevertheless, in all cases, outliers must be documented and reported along with the rest of the data.

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